Young Researchers in Extremal and Probabilistic Combinatorics

Book of Abstracts

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Organizers: Tuan Tran, Casey Tompkins.

IBS Discrete Mathematics Group
https://dimag.ibs.re.kr/

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Abstracts

Longest Cycles in Sparse Random Graphs and Where to Find Them

Michael Anastos
Freie Universität Berlin

Let $L_{c,n}$ be the length of the longest cycle in a sparse binomial random graph $G_{n,p}$, $p = c/n$, $c > 1$. Erdős conjectured that if $c > 1$ then w.h.p. $L_{c,n} \geq \ell(c)n$ for some strictly positive function on $(1, \infty)$ that is independent of n. His conjecture was proved by Ajtai, Komlós and Szemerédi and in a slightly weaker form by Fernandez de la Vega. Henceforward there has been a line of research in trying to bound $L_{c,n}$, with the best bound be given by Frieze who showed that $L_{c,n} \geq 1-(1+\epsilon_c)(1+c)e^{-c}$ where $\epsilon_c \to 0$ as $c \to \infty$. In this talk we will discuss how one can identify a set of vertices that spans a longest cycle which can be used in calculating $L_{c,n}$ and showing that $\frac{L_{c,n}}{n}$ converges to some function $f(c)$, that is independent of n, almost surely for significantly large c. This talk is based on a joint work with Alan Frieze.

Ramsey simplicity of random graphs

Simona Boyadzhiyska
Freie Universität Berlin

We say that a graph G is *q-Ramsey* for another graph H if any q-coloring of the edges of G yields a monochromatic copy of H. Much of the research related to Ramsey graphs is concerned with determining the smallest possible number of vertices in a q-Ramsey graph for a given H, known as the *q-color Ramsey number of H*. In the 1970s, Burr, Erdős, and Lovász initiated the study of another graph parameter in the context of Ramsey graphs: the minimum degree.

A straightforward argument shows that, if G is a minimal q-Ramsey graph for H, then we must have $\delta(G) \geq q(\delta(H) - 1) + 1$, and we say that H is *q-Ramsey simple* if this bound can be attained. In this talk, we will ask how typical Ramsey simplicity is; more precisely, we will discuss for which pairs p and q the random graph $G(n, p)$ is almost surely q-Ramsey simple.

This is joint work with Dennis Clemens, Shagnik Das, and Pranshu Gupta.

Mixing time and expanders

Debsoumya Chakraborti
IBS Discrete Mathematics Group

Consider regular graphs in which the random walks starting from a positive fraction of vertices have small mixing time. We establish that any such graph is virtually an expander, meaning that it becomes expander after deleting a small number of vertices. As a corollary, it shows the existence of a long cycle in such a graph, which improves a result of Pak. This talk will be based on a joint work with Jaehoon Kim, Jinha Kim, Minki Kim, and Hong Liu.

Schur's Theorem in randomly perturbed sets

Shagnik Das
National Taiwan University

A set of integers A is said to be Schur if any two-colouring of A results in monochromatic x, y and z with $x + y = z$. We study the following problem: how many random integers from $[n]$ need to be added to some $A \subset [n]$ to ensure that the resulting set is Schur with high probability? Hu showed in 1980 that when $|A| > 4n/5$, no random integers are needed, as A is already guaranteed to be Schur. Recently, Aigner-Horev and Person showed that for any dense set of integers $A \subseteq [n]$, adding $\omega(n^{1/3})$ random integers suffices, noting that this is optimal for sets A with $|A| \leq n/2$. In this talk we close the gap between these two results, showing that if $A \subset [n]$ with $|A| = n/2 + t < 4n/5$, then adding $\omega(\min(n^{1/3}, n/t))$ random integers will result in a set that is Schur with high probability, and that this is best possible for all t. We further provide some bounds for when the set A is sparse.

This is joint work with Charlotte Knierim and Patrick Morris.

Exact results for generalized Turán problems

Dániel Gerbner
Rényi Institute

The generalized Turán function $ex(n, H, F)$ is the largest number of (unlabeled) copies of H in F-free n-vertex graphs. We present some results where $ex(n, H, F)$ is determined exactly for sufficiently large n. Based on joint works with Cory Palmer and Balázs Patkós.

Degenerated generalized Turán numbers of cycles

Andrzej Grzesik
Jagiellonian University

The generalized Turán number is the largest number of copies of a graph H in an n-vertex graph not containing a graph F as a subgraph. In the talk we will discuss this problem in the case when both H and F are cycles by presenting known bounds and recent developments. We will concentrate especially on the degenerated setting, which is when the generalized Turán number is of a lower order than $n^{|H|}$. In particular, we will sketch the proof providing the first exact solution in this setting.

Extremal functions for sparse minors

Kevin Hendrey
IBS Discrete Mathematics Group

The extremal function $c(H)$ of a graph H is the supremum of densities of graphs not containing H as a minor, where the density of a graph is the ratio of the number of edges to the number of vertices. Myers and Thomason (2005), Norin, Reed, Thomason and Wood (2020), and Thomason and Wales (2019) determined the asymptotic behaviour of $c(H)$ for all polynomially dense graphs H, as well as almost all graphs of constant density. We explore the asymptotic behavior of the extremal function in the regime not covered by the above results. Joint work with Sergey Norin and David R. Wood.

Subgraph densities in minor-closed classes (and beyond)

Tony Huynh
Monash University

Many classical problems in extremal graph theory can be phrased as determining the maximum number of copies of a fixed graph H in G, where G ranges over all n-vertex graphs in some fixed graph class C. In this talk, we determine this answer precisely (up to a constant factor) for all graphs H and various minor-closed classes C (including all graphs embeddable on a fixed surface). In the special case that H is a forest, we derive stronger results that hold for many sparse graph classes C which are not minor-closed.

This is joint work with Gwenaël Joret and David Wood.

Tiling with monochromatic bipartite graphs of bounded maximum degree

Oliver Janzer
University of Cambridge

We prove that for any $r \in \mathbb{N}$, there exists a constant C_r such that the following is true. Let $\mathcal{F} = \{F_1, F_2, \dots\}$ be an infinite sequence of bipartite graphs such that $|V(F_i)| = i$ and $\Delta(F_i) \leq \Delta$ hold for all i. Then in any r-edge coloured complete graph K_n, there is a collection of at most $\exp(C_r\Delta)$ monochromatic subgraphs, each of which is isomorphic to an element of $\mathcal{F}$, whose vertex sets partition $V(K_n)$. This proves a conjecture of Corsten and Mendonça in a strong form and generalizes results on the multicolour Ramsey numbers of bounded-degree bipartite graphs. Joint work with António Girão.

Binary scalar products

Andrey Kupavskii
CNRS

Let A,B be two families of vectors in $\mathbb{R}^d$ that both span it and such that $< a, b >$ is either 0 or 1 for any a, b from A and B, respectively. We show that $|A||B| \leq (d+1)2^d$. This allows us to settle a conjecture by Bohn, Faenza, Fiorini, Fisikopoulos, Macchia, and Pashkovich (2015) concerning 2-level polytopes (polytopes such that for every facet-defining hyperplane H there is its translate H' such that H together with H' cover all vertices). The authors conjectured that for every d-dimensional 2-level polytope P the product of the number of vertices of P and the number of facets of P is at most $d2^{d+1}$, which we show to be true. We will also discuss generalizations of this result to several families. Joint work with Stefan Weltge and Fedor Noskov.

Hypergraphs with minimum uniform Turán density

Ander Lamaison
Masaryk University

Reiher, Rödl and Schacht showed that the uniform Turán density of every 3-uniform hypergraph is either 0 or at least $1/27$, and asked whether there exist 3-uniform hypergraphs with uniform Turán density equal or arbitrarily close to $1/27$. We construct 3-uniform hypergraphs with uniform Turán density equal to $1/27$. Joint work with Frederik Garbe and Dan Král.

Ramsey Properties for V-shaped Posets in the Boolean Lattices

Wei-Tian Li
National Chung Hsing University

Given posets $\mathbf{P}_1, \dots, \mathbf{P}_k$, we define the Boolean poset number $R(\mathbf{P}_1, \dots, \mathbf{P}_k)$ to be the minimum number n such that for every coloring of elements of the Boolean lattice $\mathbf{B}_n$ with k colors, there exists a poset $\mathbf{P}_i$ in $\mathbf{B}_n$ whose elements are all of color i. This function was first introduced by Axenovich and Walzer. Recently, many results on determining $R(\mathbf{Q}_m, \mathbf{Q}_n)$, where $\mathbf{Q}_m$ and $\mathbf{Q}_n$ are Boolean lattices, have being published. In this talk, we will talk about some results on $R(\mathbf{P}_1, \dots, \mathbf{P}_k)$ for each $\mathbf{P}_i$'s being the V-shaped poset. That is, a poset obtained by identifying the minimal elements of two chains.

Moreover, we define the Boolean rainbow Ramsey number $RR(\mathbf{P}, \mathbf{Q})$ the minimum number n such that for every coloring of elements of $\mathbf{B}_n$, there exists either a monochromatic $\mathbf{P}$ or rainbow $\mathbf{Q}$. The upper bound for $RR(\mathbf{P}, \mathbf{A}_k)$ was given by Chang, Li, Patkos, Gerbner, Vizer, Methuku, and Nagy for general poset $\mathbf{P}$ and k-element antichain $\mathbf{A}_k$. We will study this function for $\mathbf{P}$ being the V-shaped poset in this talk as well.

This is a joint work with Hong-Bin Chen, Wei-Han Chen, Yen-Jen Cheng, Chia-An Liu.

Maximal 3-wise intersecting families

Ben Lund
IBS Discrete Mathematics Group

A family $\mathcal{F}$ of subsets of $\{1, 2, \dots, n\}$ is called maximal k-wise intersecting if every collection of at most k members from $\mathcal{F}$ has a common element, and moreover no set can be added to $\mathcal{F}$ while preserving this property. In 1974, Erdős and Kleitman asked for the smallest possible size of a maximal k-wise intersecting family, for $k \geq 3$. We resolve this very old problem in the case $k = 3$ and n even and sufficiently large. This is joint work with Kevin Hendrey, Casey Tompkins and Tuan Tran.

Vector sum-intersection theorems

Balázs Patkós
Rényi Institute

We introduce the following generalization of set intersection via characteristic vectors: for $n, q, s, t \geq 1$ a family $\mathcal{F} \subseteq \{0, 1, \dots, q\}^n$ of vectors is said to be *s-sum t-intersecting* if for any distinct $x, y \in \mathcal{F}$ there exist at least t coordinates, where the entries of x and y sum up to at least s, i.e. $|\{i : x_i + y_i \geq s\}| \geq t$. The original set intersection corresponds to the case $q = 1, s = 2$. We address analogs of several variants of classical results in this setting: the Erdős–Ko–Rado theorem and the theorem of Bollobás on intersecting set pairs.

Joint with Zs. Tuza and M. Vizer

Pósa-type results for Berge Hypergraphs

Nika Salia
Rényi Institute

A Berge-path of length k in a hypergraph $\mathcal{H}$ is a sequence $v_1, h_1, v_2, h_2, \dots, v_k, h_k, v_{k+1}$ of distinct vertices and hyperedges with $v_{i+1} \in h_i, h_{i+1}$ for all $i \in [k]$. Füredi, Kostochka, and Luo recently gave Dirac-type minimum degree conditions that force non-uniform Hypergraphs to have Hamiltonian cycles. We give Pósa-type lower bounds for degree sequences for r-uniform and non-uniform Hypergraphs that force Hamiltonian cycles.

Small doubling, atomic structure and ℓ-divisible set families

István Tomon
ETH Zurich

Let $f(n, k, \ell)$ denote the size of a maximal family $F \subset 2^{[n]}$ such that the intersection of any k members of F has size divisible by ℓ. The famous Eventown theorem states that $f(n, 2, 2) = 2^{\lfloor n/2 \rfloor}$, and in general $f(n, k, 2) = f(n, 2, 2)$ for every $k \geq 2$. It might be tempting to conjecture that $f(n, 2, \ell) = 2^{\lfloor n/\ell \rfloor}$ in general, but this was shown to be false by Frankl and Odlyzko in 1983. On the other hand, they proposed the conjecture that $f(n, k, \ell) = 2^{\lfloor n/\ell \rfloor}$ does hold if k is sufficiently large with respect to ℓ. In a joint work with Lior Gishboliner and Benny Sudakov, we resolved this old conjecture, and in this talk I will present some of the ideas.

Existence of common graphs with large chromatic number

Jan Volec
Czech Technical University

Ramsey's Theorem states that for every graph H, there exists a number $R = R(H)$ such that all 2-edge-colorings of K_R contain a monochromatic copy of H. This talk focuses on the natural quantitative extension of this result: how many monochromatic copies are in every 2-edge-coloring of a large complete graph, and, particularly, what are those graphs H for which the edge-coloring asymptotically minimizing their monochromatic count is random-like? Such graphs H are called common.

A classical result of Goodman from 1959 states that triangle is a common graph. On the other hand, Thomason proved in 1989 that no clique of order at least four is common, and existence of a common graph with chromatic number larger than three was open until about 10 years ago, when Hatami, Hladky, Kral, Norin and Razborov proved that the 5-wheel is common. In this talk, we show that for every $k > 4$, there exists a common graph with chromatic number k.

This is a joint work with D. Kral, J. Noel, S. Norin, and F. Wei

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Constructions in combinatorics via neural networks

Adam Zsolt Wagner
Tel Aviv University

Recently, significant progress has been made in the area of machine learning algorithms, and they have quickly become some of the most exciting tools in a scientist's toolbox. In particular, recent advances in the field of reinforcement learning have led computers to reach superhuman level play in Atari games and Go, purely through self-play. In this talk I will give a quick introduction to neural networks and reinforcement learning algorithms. I will also indicate how these methods can be adapted to the "game" of trying to find a counterexample to a mathematical conjecture, and show some examples where this approach was successful.

Enumerating independent sets in Abelian Cayley graphs

Liana Yepremyan
London School of Economics

We show that any Cayley graph on an Abelian group of order $2n$ and degree $\tilde{\Omega}(\log n)$ has at most $2^{n+1}(1+o(1))$ independent sets. This bound is tight up to the o(1) term whenever the graph is bipartite. Proof techniques include the graph container method of Sapozhenko and the Plünnecke-Rusza-Petridis inequality from additive combinatorics.

This is joint work with Aditya Potukuchi.

On multicolor Ramsey numbers and subset-coloring of hypergraphs

Yelena Yuditsky
Université libre de Bruxelles

The multicolor hypergraph Ramsey number $R_k(s,r)$ is the minimal n, such that in any k-coloring of all r-element subsets of $[n]$, there is a subset of size s, all whose r-subsets are monochromatic. We present a new "stepping-up lemma" for $R_k(s,r)$: If $R_k(s,r) > n$, then $R_{k+3}(s+1,r+1) > 2^n$. Using the lemma, we improve some known lower bounds on multicolor hypergraph Ramsey numbers. Furthermore, given a hypergraph $H = (V,E)$, we consider the Ramsey-like problem of coloring all r-subsets of V such that no hyperedge of size $> r$ is monochromatic. We provide upper and lower bounds on the number of colors necessary in terms of the chromatic number $\chi(H)$. In particular, we show that this number is $O(log^{(r-1)}(r\chi(H)) + r)$, where $\log^{(m)}$ denotes m-fold logarithm.

Joint work with Bruno Jartoux, Chaya Keller and Shakhar Smorodinsky.

Zero subsums in vector spaces over finite fields

Dimitry Zakharov
Moscow Institute of Physics and Technology

The Olson constant $OL(\mathbb{F}_p^d)$ represents the minimum positive integer t with the property that every subset $A \subset \mathbb{F}_p^d$ of cardinality t contains a nonempty subset with vanishing sum. The problem of estimating $OL(\mathbb{F}_p^d)$ is one of the oldest questions in additive combinatorics, with a long and interesting history even for the case d = 1. We prove that for any fixed $d \geq 2$ and $\varepsilon > 0$, the Olson constant of $\mathbb{F}_p^d$ satisfies the inequality

$$OL(\mathbb{F}_p^d) \leq (d - 1 + \varepsilon)p$$

for all sufficiently large primes p.

Joint work with Cosmin Pohoata.

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